

Lamb Waves Propagation in Semiconductor Plates

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ABSTRACT: The present article is devoted to investigate the propagation of thermo-diffusive lamb waves in semiconductor plates. It is observed that, a shear horizontal (SH) purely transverse wave which gets decoupled from rest of the motion also propagates without dispersion and remains independent of the influence of other fields, in such materials. The secular equation for elastodiffusive (EN/EP) and thermodiffusive (TN/TP) lamb waves have been derived. The phase velocity and attenuation coefficient have been obtained. The secular equations are being solved numerically for germanium (Ge) and silicon (Si) semiconductor to obtain the profiles of phase velocity and attenuation coefficient. The waves are found to be dispersive in character and attenuated in space.

Keywords: relaxation time; electrons and holes; germanium and silicon.

INTRODUCTION: Maruszewski¹⁻² presented the theoretical consideration and developments of the simultaneous interactions of elastic, thermal and diffusion of charge carrier's fields in semiconductors. Maruszewski² formulated the problem of interaction of various fields mathematically. Some researchers³⁻⁶ modified Fourier law of heat conduction and constitutive relations so as to obtain a hyperbolic equation for heat conduction.

FORMULATION OF THE PROBLEM: We consider a homogeneous isotropic, thermoelastic semi conducting plate of thickness $2d$ initially at uniform temperature T_0 and in the undisturbed state. The basic governing equations for such a medium are taken as²

$$\mu u_{i,jj} + (\lambda + \mu) u_{i,jj} - \rho \ddot{u}_i - \lambda^n N_{,i} - \lambda^p P_{,i} - \lambda^T \theta_{,i} = 0$$

$$K \theta_{,ii} + m^{nq} N_{,ii} + m^{pq} P_{,ii} - \left(1 + t^Q \frac{\partial}{\partial t}\right) \left(\rho C_e \dot{\theta} + \rho T_0 \alpha^n \dot{N} + \rho T_0 \alpha^p \dot{P} + T_0 \lambda^T u_{k,k} \right) - \rho \left(a_1^n \dot{N} + a_1^p \dot{P} \right) = - \left(a_1^n g^n + a_1^p g^p \right)$$

$$\rho D^n N_{,ii} + m^{qn} \theta_{,ii} - \rho \left(1 - a_2^n T_0 \alpha^n + t^n \frac{\partial}{\partial t} \right) \dot{N} - a_2^n \left(\rho C_e \dot{\theta} + \rho T_0 \alpha^p \dot{P} + T_0 \lambda^T u_{k,k} \right) = \left(1 + t^n \frac{\partial}{\partial t} \right) g^n \quad (1)$$

$$\rho D^p P_{,ii} + m^{qp} \theta_{,ii} - \rho \left(1 - a_2^p T_0 \alpha^p + t^p \frac{\partial}{\partial t} \right) \dot{P} - a_2^p \left(\rho C_e \dot{\theta} + \rho T_0 \alpha^n \dot{N} + T_0 \lambda^T u_{k,k} \right) = \left(1 + t^p \frac{\partial}{\partial t} \right) g^p$$

where the following notations are used:

$$N = n - n_0, P = p - p_0, \quad a_1^n = \frac{a^{Qn}}{a^Q}, a_1^p = \frac{a^{Qp}}{a^Q},$$

$$a_2^n = \frac{a^{Qn}}{a^n}, a_2^p = \frac{a^{Qp}}{a^p} \quad (2)$$

λ, μ are the lame constants; u_i is the elastic displacement; ρ is the density of the semiconductor;

λ^n, λ^p are the elastodiffusive constants; λ^T is the thermo elastic constant; K is the coefficient of the heat conduction; $m^{nq}, m^{pq}, m^{qr}, m^{qr}$ are the Peltier-Seebeck-Dufour-Soret- like constants; t^Q, t^n, t^p are the relaxation times of heat, electron, and hole fields; C_e is the specific heat; α^n, α^p are the thermo diffusive constants: $a^{Qn}, a^{Qp}, a^Q, a^n, a^p$

are the flux like constants; D^n, D^p are the diffusions coefficients of electrons and holes. Here n, p and n_0, p_0 are respectively the non-equilibrium and equilibrium values of electrons and holes concentrations.

Upon introducing the potential functions ϕ and ψ through the relation $u = \nabla\phi + \nabla \times \psi, \nabla \cdot \psi = 0$ in equations (1) we obtain

$$\nabla^2 \phi - \ddot{\phi} - \bar{\lambda}_n N - \bar{\lambda}_p P - \theta = 0, \nabla^2 \psi = \frac{1}{\delta^2} \psi \quad (3a)$$

$$\begin{aligned} -\varepsilon_T \nabla^2 (\dot{\phi} + t^Q \ddot{\phi}) + \varepsilon^{nq} \nabla^2 N - \left\{ \frac{\alpha_0^n \partial^2}{\partial t^2} + (a_0^n + \alpha_0^n) \frac{\partial}{\partial t} + \frac{a_0^n}{t_n^+} \right\} N \\ + \varepsilon^{pq} \nabla^2 P - \left\{ \frac{\alpha_0^p \partial^2}{\partial t^2} + (a_0^p + \alpha_0^p) \frac{\partial}{\partial t} + \frac{a_0^p}{t_p^+} \right\} P + \nabla^2 \theta - (\dot{\theta} + t^Q \ddot{\theta}) = 0, \end{aligned} \quad (3b)$$

$$\begin{aligned} -\varepsilon_n \varepsilon_T \nabla^2 \dot{\phi} + \nabla^2 N - \frac{K}{\rho C_e D^n} \left[-\frac{1}{t_n^+} + \left(1 - \frac{\varepsilon_n \alpha_0^n D^n}{k} - \frac{t^n}{t_n^+} \right) \frac{\partial}{\partial t} + \frac{t^n \partial^2}{\partial t^2} \right] N \\ - \varepsilon_n \alpha_0^p \dot{P} - \varepsilon_n \dot{\theta} + \varepsilon^{qn} \nabla^2 \theta = 0, \end{aligned} \quad (3c)$$

$$\begin{aligned} -\varepsilon_p \varepsilon_T \nabla^2 \dot{\phi} + \nabla^2 P - \frac{K}{\rho C_e D^p} \left[-\frac{1}{t_p^+} + \left(1 - \frac{\varepsilon_p \alpha_0^p D^p}{k} - \frac{t^p}{t_p^+} \right) \frac{\partial}{\partial t} + \frac{t^p \partial^2}{\partial t^2} \right] P \\ - \varepsilon_p \alpha_0^n \dot{N} - \varepsilon_p \dot{\theta} + \varepsilon^{qp} \nabla^2 \theta = 0, \end{aligned} \quad (3d)$$

where we have take $\psi = (0, -\psi, 0)$ and we have defined the quantities

$$x_i' = \frac{\omega^* x_i}{c_1}, \quad t' = \omega^* t, \quad \theta' = \frac{\theta}{T_0}, \quad P' = \frac{P}{p_0}$$

$$N' = \frac{N}{n_0}, \quad u_i' = \frac{\rho \omega^* c_1}{\lambda^T T_0} u_i, \quad t^Q = t^Q \omega^*,$$

$$t^p' = t^p \omega^*, \quad t^n' = t^n \omega^*, \quad t_n^+' = t_n^+ \omega^*$$

$$t_p^+' = t_p^+ \omega^*, \quad \delta^2 = \frac{c_2^2}{c_1^2},$$

$$\varepsilon_T = \frac{\lambda^T T_0}{\rho C_e (\lambda + 2\mu)}, \quad \omega^* = \frac{C_e (\lambda + 2\mu)}{K},$$

$$c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_2^2 = \frac{\mu}{\rho}, \quad k = \frac{K}{\rho C_e}, \quad \bar{\lambda}_n = \frac{\lambda^n n_0}{\lambda^T T_0}$$

$$\bar{\lambda}_p = \frac{\lambda^p p_0}{\lambda^T T_0}, \quad \varepsilon^{qn} = \frac{m^{qn} T_0}{\rho D^n n_0}, \quad \varepsilon^{qp} = \frac{m^{qp} T_0}{\rho D^p p_0},$$

$$\begin{aligned} a_0^n = \frac{a_1^n n_0}{C_e T_0}, \quad \varepsilon_n = \frac{a_2^n K T_0}{\rho n_0 D^n}, \quad \varepsilon_p = \frac{a_2^p K T_0}{\rho p_0 D^p}, \quad \varepsilon^{pq} = \frac{m^{pq} p_0}{K T_0}, \\ \varepsilon^{nq} = \frac{m^{nq} n_0}{K T_0}, \quad a_0^p = \frac{a_1^p p_0}{C_e T_0}, \\ \alpha_0^n = \frac{\alpha^n n_0}{C_e}, \quad \alpha_0^p = \frac{\alpha^p p_0}{C_e} \end{aligned} \quad (4)$$

The dashes have been suppressed for convenience.

The non-dimensional boundary conditions on the surface $x_3 = \pm d$ are given by

$$\begin{aligned} \phi - 2\delta^2 (\phi_{,xx} + \psi_{,xz}) = 0, \\ \psi - 2\delta^2 (\psi_{,xx} - \phi_{,xz}) = 0, \\ \varepsilon^{qn} (\theta_{,3} + h_T \frac{\varepsilon_n}{\varepsilon^{qn}} \theta) + N_{,3} + h_n \bar{\varepsilon}_{nq} N = 0 \\ \varepsilon^{qp} (\theta_{,3} + h_T \frac{\varepsilon_p}{\varepsilon^{qp}} \theta) + P_{,3} + h_p \bar{\varepsilon}_{pq} P = 0 \end{aligned} \quad (5)$$

where $h_T = \frac{K^s c_1}{K\omega^*}$, $h_n = \frac{a_0^n s^n}{\varepsilon^{nq} c_1}$,

$h_p = \frac{a_0^p s^p}{\varepsilon^{pq} c_1}$, $\bar{\varepsilon}_{nq} = \frac{m^{nq}}{\rho D^n a_1^n}$, $\bar{\varepsilon}_{pq} = \frac{m^{pq}}{\rho D^p a_1^p}$.

Now we confine our discussion to the propagation of EN/EP and TN/TP waves in two dimensional semiconductor plates.

SECULAR EQUATIONS:

Elasto-diffusive Lamb Waves: Let us consider the case of the EN waves concerning the reciprocal dynamical interactions of the elastic and electron diffusion field in the plate with boundaries $x_3 = \pm d$. The thermal and hole fields are omitted ($P = \theta = 0$, $\varepsilon_T = 0 = \varepsilon^{nq}$, $\alpha_0^n = 0 = a_0^n$). The system of equations (3) with the help of appropriate boundary conditions (5) leads to the following secular equation

$$\left[\frac{T_1}{T_5} \right]^{\pm 1} - \frac{(K^2 V^2 L_1)}{(K^2 V^2 L_3)} \left[\frac{T_3}{T_5} \right]^{\pm 1} = -\frac{Q}{P} \frac{(g_1 L_3 - g_3 L_1)}{(K^2 V^2 L_3)} \quad (6)$$

Where;

$P = \delta^2 (k^2 - \beta^2)$, $Q = 2ik\delta^2 \beta$, $g_i = 2ik\delta^2 m_i$, $i = 1,3$,

$L_i = (m_i + \alpha^2)$, $i = 1,3$

The secular equation for TP waves can be written from (6) by replacing N with P and n with p.

Thermo-diffusive Lamb Wave: We now consider thermodiffusive wave we concerns the propagation of the T N waves in the plate with surfaces $x_3 = \pm d$.The system of governing equations (3) involving rest of the fields variables along with appropriate choice of boundary conditions (5) provides us

$$\left[\frac{T_1}{T_3} \right]^{\pm} = \left[-B \pm \sqrt{B^2 - 4AC} \right] / 2A \quad (7)$$

$A = m_1 m_3 [P_3 G_3 (Q_1 F_1 - P_1 G_1) + P_3 Q_3 (F_1 Q_1 - G_1 P_1)]$

$C = m_1 m_3 \{P_1 G_1 (Q_3 F_3 - G_3 P_3) + Q_1 F_1 (P_3 G_3 - Q_3 F_3)\}$

$B = \left\{ \begin{aligned} &(G_1^2 + F_1^2 m_1^2)(Q_3^2 + P_3^2 m_3^2) - (Q_1 G_1 + P_1 F_1 m_1^2)(F_3 P_3 m_3^2 + Q_3 G_3) - G_3 Q_1 (G_1 Q_3 - G_3 Q_1) \\ &+ P_3 m_3^2 Q_1 (P_3 G_1 - Q_1 F_3) + P_1 m_1 m_3 [P_3 m_1 m_3 (F_1 P_3 - P_1 F_3) - G_3 (F_1 Q_3 - G_3 P_1)] \end{aligned} \right\}$

$F_i = a_1 L_i + a_2$, $G_i = b_1 L_i + b_2$, $P_i = e_1 + h L_i$, $Q_i = g_1 + i L_i - ikv f_i$, $i = 1,3$

The secular equation (6) and (7) being secular equation contain complete information regarding wave number, frequency, phase velocity and attenuation coefficients of the lamb waves in such physical models of the plate situations. The numerical and graphically comparison is being done for germanium and we found that germanium is better conductor

then silicon. The interaction of mechanical, thermal and electron/hole charge carrier fields has attributed to significant modifications in the values of phase velocity and attenuation coefficients of elastic, thermal and diffusive waves in the low and high frequency regimes.

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